Classical Demand Theory

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Nov 12, 2008 (Afterclass version 7pm) Lecture Notes for Graduate Level Advanced Microeconomics, Peking University Guanghua School of Management

Ch. 3 is a long chapter. It contains:

We will only cover the following in this lecture:

Some topics that will be possibly covered in the next lectures:

Relationship between Demand, Indirect Utility and Expenditure Functions
Integrability Welfare
The Strong Axiom of Revealed Preference
Note: We will not cover "Duality"

→Overview
Preference Relations
Preference and Utility
The Utility Maximization Problem
The Expenditure Minimization Problem

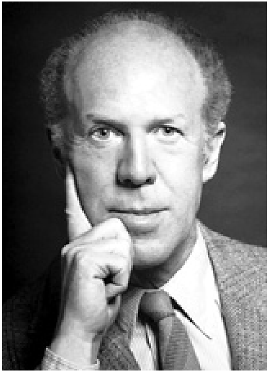
- In the last lecture, we went over a few fundamental results in economics without an explicit formulation of such functions except the use of revealed preference
- We will make the formulation of utility or preference functions explicit today

What I Expect From You in Exams

- Today we need to cover a few real analysis concepts. But do not worry if you have not done real analysis, we will rely on some results without attempting to prove them. At times the intuition of them are enough to get us through.
- In each lecture:
  - When I ask you to refer to the math appendix (especially if I have not done relevant materials in the homework or discussed in details in class), most likely I don't expect you to know how to prove them in exams. But I will still need you to know how to apply them.
  - I only go over proofs that illustrate a new technique or that the results are important. The omitted proofs could still be on exams.

Gerard Debreu

- “for having incorporated new analytical methods into economic theory and for his rigorous reformulation of the theory of general equilibrium” Press Release on 17 October 1983, The Royal Swedish Academy of Sciences.



John R Hicks

- “In his most well-known work…Value and Capital,…He presented a complete economic equilibrium model with aggregated markets for commodities, factors of production, credit and money. The construction of this model included a number of innovations, i.e., a further development of older theories of consumption and of production…” Press Release on 25 October 1972, The Royal Swedish Academy of Sciences.

Setting

- L commodities
- Commodity bundle (vector): $x = (x_1, ..., x_L) \in \mathbb{R}^L_+$
- $\succsim$ is rational (complete and transitive)
- Consumption set: unless stated otherwise $X \subseteq \mathbb{R}^L_+$ is the set of all feasible commodity bundles
- Price vector: $p = (p_1, ..., p_L) \in \mathbb{R}^L_{++}$

- Definition 3.B.2 $\succsim$ on X is monotone iff $x \in X$ and $y >> x \implies y \succ x$. It is strongly monotone if $y \geq x$ and $y \neq x \implies y \succ x$.
- Comments:
  - FD and at least one commodity is a good (not bad) together imply monotonicty.
  - All commodities are goods iff there is strong monotonicity. Else if it is only the bads that are increased, even with FD, you can't be strictly better off.
  - In any case, X can be redefined to remove bads. For example, x_1 being garbage can be renamed as "the absence of garbage".
  - Alternatively, one could use weak monotonicity (as in MWG Ex. 3.B.2 or Varian, which is consistent with the verbal description in MWG not in Definition 3.B.2) as $y \geq x \implies y \succsim x$, then either FD (irrespective of all commodities being bad) or all commodities are goods is sufficient for weak monotonicity.

- Definition 3.B.3 $\succsim$ on X is locally nonsatiated iff $\forall x \in X$ and $\epsilon > 0, \exists y \in X$ s.t. $||y - x|| \leq \epsilon$ and $y \succ x$.
- Comments: Rule out "thick" indifference curve
- Question: If we don't rule out the case that all commodities are "bads", do we have local nonsatiation?
- Homework MWG 3.B.1 Show that monotonicity implies local nonsatiation.

- Definition 3.B.4 $\succsim$ on X is convex iff $\forall x \in X$, the upper contour set $\{y \in X : y \succsim x\}$ is convex; that is, if $y \succsim x$ and $z \succsim x$ then $\forall \alpha \in [0,1], \alpha y + (1-\alpha) z \succsim x$.
- Comments:
  - A central assumption in economics
  - Related to the diminishing marginal rates of substitution.
  - Can be made possible by some sort of aggregation (Ch. 4)
- Question: Give an example in which your preference is not convex.

- Definition 3.B.5 $\succsim$ on X is strictly convex iff $\forall x \in X$ if $y \succsim x, z \succsim x$ and $y \neq z$ then $\forall \alpha \in (0,1), \alpha y + (1-\alpha) z \succ x$.

- Examples of preferences in which one could deduce the entire preference relation from a single indifference set

- Example 1
  - Definition 3.B.6 A monotone $\succsim$ on $X = \mathbb{R}^L_+$ is homothetic iff this holds: if $x \sim y$, then $\forall \alpha \in [0, \infty) : \alpha x \sim \alpha y$.
    - Alternative definition: A homothetic function is a monotone function, which transforms a function that is homogeneous of degree 1
    - Note: A monotone function is a strictly increasing function.
    - Question: Contrast homothetic and homogenous function

- Example 2
  - Definition 3.B.7 $\succsim$ on $X = (-\infty, \infty) \times \mathbb{R}_{+}^{L-1}$ is quasilinear wrt the numeraire commodity (say commodity 1) iff
    - If $x \sim y$, then for $e_1 = (1, 0, ..., 0)$ and $\forall \alpha \in (-\infty, \infty) : x + \alpha e_1 \sim y + \alpha e_1$
    - Homework MWG 3.D.4 illustrates why $\alpha \in (-\infty, \infty)$.
    - Good 1 is desirable, that is, $\forall \alpha > 0, \forall x : x + \alpha e_1 \succ x$.

- Definition 3.C.1 $\succsim$ on X is continuous iff for all sequence of pairs $\{(x^n, y^n)\}_{n=1}^{\infty}$ with $x^n \succsim y^n$ for all n, we have $x \succsim y$ where $x = \lim_{n\to\infty} x^n$ and $y = \lim_{n\to\infty} y^n$
  - That is, preference is preserved under limits.

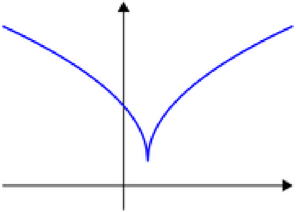
- Proposition 3.C.1.A $\succsim$ on X is continuous (in the sense of Definition 3.C.1) $\iff$ the upper contour set $\{y \in X : y \succsim x\}$ and lower contour set $\{y \in X : x \succsim y\}$ are closed
- Proof:
  - "$\implies$": A set is closed iff every convergent sequence in that set converges to a point in that set. The closedness of the lower contour set is ensured if we let $x^n = x$ for all n. Similarly, we can prove closedness of the upper contour set.
  - "$\impliedby$": You are asked to prove it in the OPTIONAL homework question MWG 3.C.3

- Proposition 3.C.1 $\succsim$ on $X = \mathbb{R}^L_+$ is rational, convex and continuous $\implies \exists$ a continuous $u(x)$ that represents $\succsim$
- Comments:
  - Any strictly increasing but discontinuous transformations of a continuous $u(x)$ also represent $\succsim$
  - Lexicographic preference relation satisfies rationality but violates continuity. It can't be represented by a utility function.

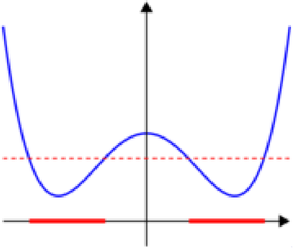
Proposition 3.C.1.B: Monotonicity and convexity of $\succsim \implies$ all utility functions representing $\succsim$ are increasing and quasiconcave.
Proof: Check MWG p.49-50 yourself.

- Definition: A function $f(x)$ is quasiconvex iff for any constant $k, S^{\leq} \equiv \{x|f(x) \leq k\}$ is a convex set. Or A function $f(x) : S \longrightarrow \mathbb{R}$ is quasiconvex iff $\forall x^{'}, x^{''} \in S, f(\alpha x^{'} + (1-\alpha)x^{''}) \leq Max\{f(x'), f(x'')\}$
- Definition: A function $f(x)$ is quasiconcave iff for any constant $k, S^{\geq} \equiv \{x|f(x) \geq k\}$ is a convex set. Or A function $f(x) : S \longrightarrow \mathbb{R}$ is quasiconcave iff $\forall x^{'}, x^{''} \in S, f(\alpha x^{'} + (1-\alpha)x^{''}) \geq Min\{f(x'), f(x'')\}$

Example 1: A quasiconvex function which is not convex.



Example 2: A function which is not quasiconvex: the set of points in the domain of the function for which the function values are below the dashed red line is the union of the two red intervals, which is not a convex set.



More comments

- Utility is an ordinal concept
  - An increasing transformation of the utility function also maintains the order of the preference it represents
  - The Cobb-Douglas Utility Function ($u(x_1, x_2) = kx_1^{\alpha}x_2^{1-\alpha}$ for $k > 0$ and $\alpha \in (0, 1)$) is hard to deal with but we can do an increasing transformation through a log function ($\alpha \ln x_1 + (1 - \alpha) \ln x_2$) that makes it easy to differentiate in the maximization problem.)
  - The difference of two utility values does not mean much when compared to the difference of another two utility values
    - If $u(a) - u(b) = 4$ and $u(c) - u(d) = 2$, does it mean consuming a over b is twice as happy as consuming c over d?
- Increasingness and quasiconcavity are ordinal properties of utility functions
- Continuity is an cardinal property of utility function

Proposition 3.C.1 $\succsim$ on $X = \mathbb{R}^L_+$ is rational, monotone and continuous $\implies$ $\exists$ a continuous $u(x)$ that represents $\succsim$
Comments:

- It is a main result in this chapter
- Debreu (1964) provides a more general treatment.

Proposition 3.C.1 $\succsim$ on $X = \mathbb{R}^L_+$ is rational, monotone and continuous $\implies \exists$ a continuous $u(x)$ that represents $\succsim$
Proof:

- Denote the diagonal ray (the locus of vectors with all L components equal) in $\mathbb{R}^L_+$ by Z
- Denote the vector with all L components equal to 1 by e
- Then $\forall \alpha \geq 0, \alpha e \in Z$
- $\forall x \in \mathbb{R}^L_+, x \geq 0$, we have $x \succsim 0$ by monotonicity
- $\forall \bar{\alpha} \geq 0$ s.t. $\bar{\alpha} e >> x$, we have $\bar{\alpha} e \succsim x$ by monotonicity
- Lemma 1: Continuity and Monotonicity $\implies \exists$ a unique value $\alpha(x) \in [0, \bar{\alpha}]$ s.t. $\alpha(x) e \sim x$.
- Rename $\alpha(x)$ as $u(x)$.
- What's left to prove are continuity and representation
- Lemma 2 (continuity): The proof of continuity is more advanced, which we will omit
- Lemma 3 (representation): $[\alpha(x) \geq \alpha(y) \iff x \succsim y]$

Proof of Lemma 3 (representation): $[\alpha(x) \geq \alpha(y) \iff x \succsim y]$

- "$\Longrightarrow$":
  - $\alpha(x) \geq \alpha(y) \implies \alpha(x)e \geq \alpha(y)e \implies \alpha(x)e \succsim \alpha(y)e$ by monotonicity.
  - By Lemma 1, we have $\alpha(x)e \sim x$ and $\alpha(y)e \sim y$.
  - Hence, $x \succsim y$ by transitivity
- "$\Longleftarrow$":(I proved it differently than MWG)
  - Given that $x \succsim y$.
  - Suppose otherwise that $\alpha(x) < \alpha(y)$. Then $\alpha(x)e << \alpha(y)e$. By monotonicity, $\alpha(y)e \succ \alpha(x)e$.
  - By Lemma 1, we have $\alpha(x)e \sim x$ and $\alpha(y)e \sim y$.
  - $y \succ x$ by transitivity. Hence $x \not\succsim y$. A contradiction.
  - We must have $\alpha(x) \geq \alpha(y)$
  - Comment on the proof in MWG: $\alpha(x)e \succsim \alpha(y)e \implies \alpha(x) \geq \alpha(y)$ doesn't follow directly from the definition of monotonicy, it needs a few-line proof.

Proof of Lemma 1: Continuity and Monotonicity $\implies \exists$ a unique value $\alpha(x) \in [0, \bar{\alpha}]$ s.t. $\alpha(x)e \sim x$.
(I added more background explanations of real analysis to the proof in MWG)

- By continuity, the upper contour set $\{y \in X : y \succsim x\}$ and lower contour set $\{y \in X : x \succsim y\}$ of x are closed
- Hence, $A^+ = \{\alpha \in \mathbb{R}_+ : \alpha e \succsim x\}$ and $A^- = \{\alpha \in \mathbb{R}_+ : x \succsim \alpha e\}$ are closed because
  - $\forall \alpha^n$ s.t. $\alpha^n e \succsim x$ we must have $\hat{\alpha} e \succsim x$ where $\hat{\alpha} = \lim_{n \to \infty} \alpha^n$ because $\{\alpha^n e\}$ is a convergent sequence in the upper contour set. By closedness of the upper contour set, every convergent sequence has to converge inside the upper contour set and thus $\hat{\alpha}$ is in A^+. Similarly, we can show that A^- is also closed.
- A^- is nonempty because it includes 0 since $\forall x \in \mathbb{R}^L_+, x \geq 0$, we have $x \succsim 0$ by monotonicity
- A^+ is nonempty because $\forall x \in \mathbb{R}^L_+, \exists \alpha^*$ s.t. $\alpha^* e >> x$. So by monotonicity, $\alpha^* e \succsim x$. Hence, $\alpha^* \in A^+$
  - For example, set $\alpha^* =$ the maximum element in $x + \epsilon, \forall \epsilon > 0$.

Proof of Lemma 1: (Continued)

- By completeness of $\succsim$, $\forall \alpha \in \mathbb{R}_+$, $\exists y$ s.t. $\alpha e \succsim y$ or $y \succsim \alpha e$, hence α is in A^+ or A^-. Thus $\mathbb{R}_+ \subseteq (A^+ \cup A^-)$
- But $(A^+ \cup A^-) \subseteq \mathbb{R}_+$ because $\alpha \in \mathbb{R}_+$. Hence $(A^+ \cup A^-) = \mathbb{R}_+$.
- Definition: X is connected iff it is NOT a union of two non-empty, disjoint, closed sets.
- Nonemptiness of A^+ and A^- and $(A^+ \cup A^-) = \mathbb{R}_+$ imply, by connectedness of $\mathbb{R}_+$, that A^+ and A^- are not disjoint $(A^+ \cap A^- \neq \emptyset)$. Hence, $\alpha e \sim x$
- (Fact: $\mathbb{R}_+$ is connected)
- α is unique because by montonicity, $\alpha_1 > \alpha_2 \implies \alpha_1(x)e \succ \alpha_2(x)e$. Both $\alpha_1(x)e$ and $\alpha_2(x)e$ cannot be indifferent to x at the same time. QED

Lexicographical preference in $X = \mathbb{R}^2_+$ says that $x \succsim y$ iff either "$x_1 > y_1$" or "$x_1 = y_1$ and $x_2 \geq y_2$".
Claim: $\succsim$ can't be represented by a utility function
Proof 1:

- No two distinct bundles are indifferent
- Thus, indifference sets are singletons
- Each point in the two dimensional $\mathbb{R}^2_+$ is an indifference set. But we need to map $\mathbb{R}^2_+$ to a one dimensional line of utility values. A mathematical impossibility.

Lexicographical preference in $X = \mathbb{R}^2_+$ says that $x \succsim y$ iff either "$x_1 > y_1$" or "$x_1 = y_1$ and $x_2 \geq y_2$".
Claim: $\succsim$ can't be represented by a utility function
Proof 2 (more rigorous):

- The rationals are a dense subset of the real numbers: every real number has rational numbers arbitrarily close to it.
- So between $u(x_1, 2)$ and $u(x_1, 1)$, there exists an $r(x_1)$ strictly between them, meaning
- $u(x_1, 2) > r(x_1) > u(x_1, 1)$
- Since $u(x_1, 1) > u(x_1^{'}, 2)$ if $x_1 > x_1^{'}$.
- Again by denseness, $u(x_1^{'}, 2) > r(x_1^{'}) > u(x_1^{'}, 1)$
- By transitivity, $r(x_1) > r(x_1^{'})$ whenever $x_1 > x_1^{'}$
- Recall that a function $g : A \rightarrow B$ is injective if $f(x) = f(y) \implies x = y$ (equivalently, $x \neq y \implies f(x) \neq f(y)$)
- So $r(\cdot)$ is an injective function from the set of real numbers (which is uncountable) to the set of rational numbers (which is countable). A mathematically impossibility.

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Utility Maximization Problem (UMP)

$$\underset{x \geq 0}{Max}\ u(x)\ s.t.\ p \cdot x \leq w \tag{1}$$

Assumptions:

- Preference is rational, continuous and locally nonsatiated
- Utility function is continuous and twice continously differentiable
- $p >> 0$
- $w > 0$
- $X = \mathbb{R}^L_+$
- The solution $x(p, w)$ to the UMP is called the Walrasian Demand Correspondence

- Proposition 3.D.1 If $p >> 0$ and $u(\cdot)$ is continuous, then the UMP has a solution.
- Proof
  - By the Weierstrass Theorem (see M.F. of the Math Appendix), a continuous function on a compact set achieves a maximum.
  - A compact set is defined as closed and bounded. Note: A set S of real numbers is called bounded from above if there is a real number k such that $k \geq s$ for all s in S. The number k is called an upper bound of S. The terms bounded from below and lower bound are similarly defined. A set S is bounded if it has both upper and lower bounds.
  - Homework: Prove that the Walrasian budget set is closed and bounded.

- Proposition 3.D.2 If $u(\cdot)$ is continuous representing a locally nonsatiated $\succsim$ on $X = \mathbb{R}^L_+$, then $x(p, w)$ satisfies
    - Homoegeneity of degree zero
    - Walras' law: $p \cdot x = w$ for all $x \in x(p, w)$
    - Convexity if $\succsim$ is convex
    - Uniqueness if $\succsim$ is strictly convex

Proof of homoegeneity of degree zero

- The budget under (p, w) are the same as under $(\alpha p, \alpha w)$.

Proof of the Walras' law

- Suppose otherwise that $p \cdot x < w$ for some $x \in x(p, w)$. Then by local nonsatiation, $\exists y$ s.t. $p \cdot y < w$ and $y \succ x$, violating x being a maximizer.

Proof of Convexity

- By Proposition 3.C.1.B: Monotonicity and convexity of $\succsim \implies$ all utility functions representing $\succsim$ are increasing and quasiconcave.
- Quasiconcavity means that $\forall \alpha \in [0,1], u(\alpha x^{'} + (1-\alpha)x^{''}) \geq \min(u(x^{'}), u(x^{''}))$
- Suppose $x^{'}, x^{''} \in x(p,w)$. Then $u(x^{'}) = u(x^{''}) \equiv u^{*}$
- We need to show that $(\alpha x^{'} + (1-\alpha)x^{''}) \in x(p,w)$
- By quasiconcavity, $\forall \alpha \in [0,1], u(\alpha x^{'} + (1-\alpha)x^{''}) \geq u^{*}$
- $p \cdot (\alpha x^{'} + (1-\alpha)x^{''}) \leq w$ because both $x^{'}$ and $x^{''}$ are affordable.
- Hence, $(\alpha x^{'} + (1-\alpha)x^{''}) \in x(p,w)$.

Proof of uniqueness

- Note that Proposition 3.C.1.B also holds for strict convexity and strict quasiconcavity.
- By strict quasiconcavity, similar to the proof of convexity, we can show that $(\alpha x^{'} + (1-\alpha)x^{''})$ is affordable and $\forall \alpha \in (0,1) : u(\alpha x^{'} + (1-\alpha)x^{''}) > u^*$
- But this contradicts $x^{'}$ and $x^{''}$ being two maximizers. So there must be only one maximizer.

Optimization

- Exact techniques used in the Firm's PMP discussed in earlier lecture notes.

- Lagrangian: $L = \ u(x) - \lambda(p \cdot x - w) + \sum_l \mu_l x_l$
- Kuhn-Tucker conditions:
  - first-order condition: $\lambda p_l = \frac{\partial u(x^*)}{\partial x_l} + \mu_l$,for $l = 1, ..., L, x^* \in x(p, w)$ and $\lambda, \mu_l \geq 0$
  - $\frac{\partial u(x^*)}{\partial x_l} \leq \lambda p_l$ for $l = 1, ..., L$, with equality if $x_l > 0$ because of the complementary slackness condition $\mu_l x_l = 0$
  - With equality above, we have $\lambda p = \nabla u(x^*)$
  - Other complementary slackness conditions: $\lambda(p \cdot x - w) = 0$ and $\mu_l x_l = 0$ for all l.

- The FOCs are not only necessary but sufficient conditions when $u(\cdot)$ is quasiconcave, monotone and has $\nabla u(x^*) \neq 0$ for all $x \in \mathbb{R}^L_+$.
- Proof: Omitted. See Math Appendix M.K.

- From the Kuhn-Tucker conditions, we obtained a relationship between the price ratio $\frac{p_l}{pk}$ and marginal rate of substitutions between l and k
  - $\frac{p_l}{p_k} = \frac{\partial u(x^*)}{\partial x_l} / \frac{\partial u(x^*)}{\partial x_k}$ if we have interior solutions for both goods. This equality will not hold if we have a corner solution.

- λ, the shadow price of relaxing the wealth constraint, is the marginal utility value of weath at the optimum because:
  - Consider the simpler case of $x_l > 0, \forall l$
  - By the chain rule

$$\frac{\partial u(x^*)}{\partial w} = \nabla u(x^*(p, w)) \cdot D_w x(p, w) \tag{2}$$

  - By $\lambda p = \nabla u(x^*)$,

$$\begin{aligned} \frac{\partial u(x^*)}{\partial w} &= \lambda p \cdot D_w x(p, w) \qquad (3) \\ &= \lambda \qquad (4) \end{aligned}$$

  - The last equality holds because $p \cdot D_w x(p, w) = 1$ since $p \cdot x(p, w) = w$ by the Walras' law.

Indirect Utility Function

- For each $(p, w) >> 0$, the indirect utility function $v(p, w)$ is defined to be equal to $u(x^*)$ where $x^* \in x(p, w)$

- Proposition 3.D.3 If $u(\cdot)$ is continuous representing a locally nonsatiated $\succsim$ on $X = \mathbb{R}^L_+$, then $v(p, w)$ satisfies
    - Homogeneity of degree zero
    - Strictly increasing in w and nonincreasing in p_l for any l
    - Quasiconvex (i.e. the negative of $v(p, w)$ is quasiconcave)
    - Continuous in p and w
- Proof: Quasiconvexity can be seen graphically for $L = 2$ (see Figure 3.D.5). For other proofs, read MWG p.56-57 yourself. Some of which might be homework questions.

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The Expenditure Minimization Problem (EMP)

$$\underset{x \geq 0}{Min}\ p \cdot x \ s.t.\ u(x) \geq u \tag{5}$$

Assumptions:

- $p >> 0$
- $u(\cdot)$ is continuous, twice continously differentiable, and represents a locally nonsatiated $\succsim$ on $X = \mathbb{R}_+^L$
- $u > u(0)$
- The minimized expenditure $e(p, u)$ for this EMP is called the expenditure function
- The solution to the EMP is called the Hicksian (or compensated) demand correspondence $h(p, u)$
- Homework Ex. 3.E.1 asks you to find the FOCs of this EMP.

- Proposition 3.E.2 If $u(\cdot)$ is continuous representing a locally nonsatiated $\succsim$ on $X = \mathbb{R}^L_+$, then $e(p, u)$ satisfies
  - Homogeneity of degree one in p
  - Strictly increasing in u and nondecreasing in p_l for any l
  - Concave in p
  - Continuous in p and u
- Proof: Read MWG p.59-60 yourself. Some of which might be homework questions.

- Proposition 3.E.1 If $u(\cdot)$ is continuous representing a locally nonsatiated $\succsim$ on $X = \mathbb{R}^L_+$, and $p >> 0$ we have:
    - $x^* \in x(p, w)$ and $w > 0$ implies that $x^* \in h(p, u)$. The minimized expenditure is exactly w.
    - $x^* \in h(p, u)$ and $u > u(0)$, then $x^* \in x(p, w)$ when $w = p \cdot x^*$. The maximized utility level is exactly u.
- Proof: Read MWG p.58-59 yourself.

Proposition 3.E.3 If $u(\cdot)$ is continuous representing a locally nonsatiated $\succsim$ on $X = \mathbb{R}^L_+$, then $\forall p >> 0$, $h(p, u)$ satisfies:

- Homogeneity of degree zero in p
    - No excess utility: For any $x \in h(p, u), u(x) = u$
    - Convexity if $\succsim$ is convex
    - Uniqueness if $\succsim$ is strictly convex
- Proof: Read MWG p.61 yourself. Some of which might be homework questions.

- Proposition 3.E.1 allows us to express

$$e(p, v(p, w)) = w \tag{6}$$

$$v(p, e(p, u)) = u \tag{7}$$

$$h(p, u) = x(p, e(p, u)) \tag{8}$$

$$x(p, w) = h(p, v(p, w)) \tag{9}$$

Hicksian Wealth Compensation

- As prices vary, $h(p, u)$ gives the level of demand that would arise if wealth were simulataneously adjusted to keep the utility level at u.
- If (p, w) is changed to (p', w),where $p_1' = p_1$ and $p_2' > p_2$, then the Hicksian wealth compensation $\triangle w_{Hicks} = e(p', u) - w$.
- See Figure 3.E.4

Hicksian Demand and the Compensated Law of Demand

Proposition 3.E.4 If $u(\cdot)$ is continuous representing a locally nonsatiated $\succsim$ on $X = \mathbb{R}^L_+$, then $\forall p >> 0$, a single-valued $h(p, u)$ satisfies the compensated law of demand:

$$\forall p', p'' : (p' - p'') \cdot [h(p'', u)] - h(p', u)] \leq 0$$

- Proof: By optimality of $h(p, u)$, we have

  $$p'' \cdot h(p'', u) \leq p'' \cdot h(p', u),$$

  and

  $$p' \cdot h(p'', u) \geq p' \cdot h(p', u)$$

- Subtracting these two inequalities yields the results.

We will not cover MWG 3.F (Duality) for this course.