

General Equilibrium with Uncertainty

Benjamin Chiao

Dec 24, 2008 (1pm, afterclass version) Lecture Notes for Graduate Level Advanced Microeconomics, Peking University Guanghua School of Management

- ▶ Overview
- ▶ State Contingent Variables: Arrow-Debreu Equilibrium
- ▶ Sequential Trade: Radner Equilibrium
- ▶ Assets with Complete Markets
- ▶ Assets with Incomplete Markets
- ▶ Summary of the Course, Issues about Debates, Exams and Others

- ▶ →Overview
- ▶ State Contingent Variables: Arrow-Debreu Equilibrium
- ▶ Sequential Trade: Radner Equilibrium
- ▶ Assets with Complete Markets
- ▶ Assets with Incomplete Markets
- ▶ Summary of the Course, Issues about Debates, Exams and Others

- ▶ This chapters incorporates uncertainty into the general equilibrium framework, which brings many applications in the fields of finance and “incomplete markets” .

Roy Radner (1927–)

- ▶ A student of Jacob Marschak
- ▶ Currently at NYU's economics and information systems departments
- ▶ Incorporated information and uncertainty into Neo-Walrasian general equilibrium theory
- ▶ He developed an “equilibrium of plans, prices and price expectations” in a sequential economy - also known as a “Radner Equilibrium” - which has since found many applications in the theory of general equilibrium, finance and incomplete markets.



- ▶ Overview
- ▶ →State Contingent Variables: Arrow-Debreu Equilibrium
- ▶ Sequential Trade: Radner Equilibrium
- ▶ Assets with Complete Markets
- ▶ Assets with Incomplete Markets
- ▶ Summary of the Course, Issues about Debates, Exams and Others

Settings

- ▶ L physical commodities
- ▶ I consumers
- ▶ J firms
- ▶ Technologies, endowments, and preferences are uncertain, which depend on the states: $s = 1, \dots, S$.
- ▶ For simplicity, each share of profits θ_{ji} remains state independent

Settings

- ▶ $(x_{1si}, \dots, x_{Lsi})$ is a state-contingent commodity vector, will be received (or delivered if it's negative) by individual i when state s occurs
 - ▶ Summarizing the s states:
 $x_i = (x_{11i}, \dots, x_{L1i}, \dots, x_{1Si}, \dots, x_{LSi}) \in \mathbb{R}^{LS}$
 - ▶ Similarly, we have i 's endowment:
 $\omega_i = (\omega_{11i}, \dots, \omega_{L1i}, \dots, \omega_{1Si}, \dots, \omega_{LSi}) \in \mathbb{R}^{LS}$
 - ▶ And j 's technology:
 $y_j = (y_{11j}, \dots, y_{L1j}, \dots, y_{1Sj}, \dots, y_{LSj}) \in \mathbb{R}^{LS}$
- ▶ i 's Preference: $x_i \succsim_i x'_i \iff \sum_s \pi_{si} u_{si}(x_{1si}, \dots, x_{Lsi}) \geq \sum_s \pi_{si} u_{si}(x'_{1si}, \dots, x'_{Lsi})$, where π_{si} is the (subjective or objective) probability of state s .
- ▶ A market exists for each contingent commodity, so p'_{ls} exist. Note that payment is not contingent.

Arrow-Debreu Equilibrium

Using this idea of state-contingent commodities, the Walrasian equilibrium developed in earlier chapters also applies with no essential modifications but the equilibrium is now called the Arrow-Debreu equilibrium. The welfare theorems also apply.

Arrow-Debreu Equilibrium

- Definition 19.C.1 An allocation $(x_1^*, \dots, x_I^*, y_1^*, \dots, y_J^*)$ and $p = (p_{11}^*, \dots, p_{LS}^*)$ is an Arrow-Debreu Equilibrium iff (note: the following is the same as Def 16.B.3)
- Profit maximization: For all j , $p \cdot y_j \leq p \cdot y_j^*$, $\forall y_j \in Y_j$.
 - Choice optimality: For all i , x_i^* is the most preferred within the budget set

$$B_i(p) = \{x_i \in X_i : p \cdot x_i \leq p \cdot \omega_i + \sum_j \theta_{ij} p \cdot y_j^*\} \quad (1)$$

- Feasibility:

$$\sum_i x_i^* = \sum_i \omega_i + \sum_j y_j^* \quad (2)$$

Criticisms of the Arrow-Debreu Equilibrium

- ▶ All trade takes place simultaneously and before the uncertainty is resolved is unrealistic

- ▶ Overview
- ▶ State Contingent Variables: Arrow-Debreu Equilibrium
- ▶ →Sequential Trade: Radner Equilibrium
- ▶ Assets with Complete Markets
- ▶ Assets with Incomplete Markets
- ▶ Summary of the Course, Issues about Debates, Exams and Others

Sequential Trade

- ▶ Consider a simple case of an exchange economy where the “forward market” is set up as:
 - ▶ Two dates: $t = 0, 1$
 - ▶ No consumption at $t = 0$
 - ▶ Uncertainty resolved at $t = 1$, when goods traded at $t = 0$ are delivered at $t = 1$
 - ▶ $x_i \in \mathbb{R}_+^{LS}$
- ▶ Denote (x_1^*, \dots, x_I^*) and $(p_{11}^*, \dots, p_{LS}^*)$ an Arrow-Debreu equilibrium.
- ▶ Suppose now a “spot market” is added, where commodities are traded at $t = 1$ also
- ▶ Question: Will there be any incentives to trade in the spot market?

- ▶ What if some contingent commodities are not available at $t = 0$ but spot market exists at $t = 1$?
 - ▶ Arrow (1953): Still pareto optimal.
 - ▶ Intuition:
 - ▶ If spot trade can occur within each state, then the only task remaining at $t = 0$ is to transfer the consumer's overall purchasing power efficiently across states.
 - ▶ This can be accomplished using contingent trade in a single commodity.
 - ▶ Such procedure reduces the number of required forward markets from LS to S .

Radner Equilibrium

Definition 19.D.1 (A) This is a Radner Equilibrium:

- ▶ $q = (q_1, \dots, q_S) \in \mathbb{R}^S$, the price vector for the **first** contingent commodity at $t = 0$
- ▶ $p_s = (p_{11}^*, \dots, p_{Ls}^*) \in \mathbb{R}^L$, the **correctly (rationally) expected** spot price vector
- ▶ $z_i'^* = (z_{1i}'^*, \dots, z_{Si}'^*) \in \mathbb{R}^S$, the consumption plans made $t = 0$ for consumption at $t = 1$. He sells short if $z_{si}'^* < 0$.
- ▶ $x_i^* = (x_{1i}^*, \dots, x_{Si}^*) \in \mathbb{R}_+^{LS}$, the consumption at $t = 1$

iff

- ▶ $\forall i : z_i'^*, x_i^*$ solves

$$\text{Max}_{x_i, z_i} U_i(x_i) \quad (3)$$

$$\text{s.t. } i) \sum_s q_s z_{si}' \leq \sum_s q_s \omega_{1si} \quad (4)$$

$$ii) \forall s : p_s \cdot x_{si} \leq \sum_{l \neq 1} p_{ls} \omega_{l1} + p_{1s} z_{s1}' \quad (5)$$

- ▶ $\forall s : \sum_i x_{si}^* + \sum_i z_{si}'^* \leq \sum_i \omega_{si}$

By letting $z'_{si} = z_{si} + \omega_{1si}$, we can write the Radner equilibrium in simpler terms:

Radner Equilibrium

Definition 19.D.1 (B) This is a Radner Equilibrium:

- ▶ $q = (q_1, \dots, q_S) \in \mathbb{R}^S$, the price vector for the **first** contingent commodity at $t = 0$
- ▶ $p_s = (p_{11}^*, \dots, p_{L_s}^*) \in \mathbb{R}^L$, the **correctly (rationally) expected** spot price vector
- ▶ $z_i^* = (z_{1i}^*, \dots, z_{S_i}^*) \in \mathbb{R}^S$, the consumption plans made $t = 0$ for consumption at $t = 1$
- ▶ $x_i^* = (x_{1i}^*, \dots, x_{S_i}^*) \in \mathbb{R}_+^{L_s}$, the consumption at $t = 1$

iff

- ▶ $\forall i : z_i^*, x_i^*$ solves

$$\text{Max}_{x_i, z_i} U_i(x_i) \quad (6)$$

$$\text{s.t. } i) \sum_s q_s z_{si} \leq 0 \quad (7)$$

$$ii) \forall s : p_s \cdot x_{si} \leq p_s \cdot \omega_{si} + p_{1s} z_{s1} \quad (8)$$

- ▶ $\forall s : \sum_i z_{si}^* \leq 0$ and $\sum_i x_{si}^* \leq \sum_i \omega_{si}$

Proposition 19.D.1 shows that the Arrow-Debreu Equilibrium is identical to the Radner Equilibrium

Proof: Skipped.

- ▶ Overview
- ▶ State Contingent Variables: Arrow-Debreu Equilibrium
- ▶ Sequential Trade: Radner Equilibrium
- ▶ →Assets with Complete Markets
- ▶ Assets with Incomplete Markets
- ▶ Summary of the Course, Issues about Debates, Exams and Others

Assets with Complete Markets

- ▶ Definition 19.E.1 A unit of asset, or security, is a title to receive an amount r_s of a numeraire (good 1) at date $t = 1$ if state s occurs. An asset is $r = (r_1, \dots, r_S) \in \mathbb{R}^S$.
- ▶ Examples:
 - ▶ $r = (1, \dots, 1)$ is a riskless asset if $L = 1$.
 - ▶ $r = (0, \dots, 0, 1, 0, \dots, 0)$ is an Arrow security used in Section 19.D
 - ▶ $r(c) = (\text{Max}\{0, r_1 - c\}, \dots, \text{Max}\{0, r_S - c\})$ is a derivative asset, where c is a strike price through a call option to buy a primary asset of return r_s when state s occurs.

With the generalization using assets, Proposition 19.E.2 proves that the Arrow-Debreu Equilibrium is identical to the Radner Equilibrium as long as the market is complete:

- ▶ A market is complete if the effective (in the sense of linear independence) number of assets at least equals to the number of states.
- ▶ Proof: Skipped.

- ▶ Overview
- ▶ State Contingent Variables: Arrow-Debreu Equilibrium
- ▶ Sequential Trade: Radner Equilibrium
- ▶ Assets with Complete Markets
- ▶ →Assets with Incomplete Markets
- ▶ Summary of the Course, Issues about Debates, Exams and Others

Assets with Incomplete Markets

- ▶ There are many situations in which incomplete markets lead to pareto inoptimality in equilibrium
- ▶ Examples
 - ▶ Sunspots Equilibrium
 - ▶ Expected utility framework
 - ▶ Probability estimates of different states is the same across consumers
 - ▶ States do not affect fundamentals of the economy
 - ▶ But the Radner equilibrium is Sunspot if it gives rise to assignment of different consumption across states via differences in expectations
 - ▶ Say if there are no assets, it can be shown that each element of the state-dependent price vector in a Radner equilibrium coincides with the price vector in a Walrasian equilibrium. When there are multiple Walrasian equilibria, if the wrong state-dependent price vector (differs from the WE price vector) turns out to be self-fulfilled, it can't be a WE price vector and thus the allocation is not necessarily PO (recall what SWT needs).

- ▶ Overview
- ▶ State Contingent Variables: Arrow-Debreu Equilibrium
- ▶ Sequential Trade: Radner Equilibrium
- ▶ Assets with Complete Markets
- ▶ Assets with Incomplete Markets
- ▶ →Summary of the Course, Issues about Debates, Exams and Others

Revised Debate Propositions

1. Risk or uncertainty as parameters of a theory is not consistent with Popper's view of philosophy of science
2. Anything can be viewed as public goods
3. It is of little value to compare the choice of telescopes with the testing of economic theories
4. Welfare economics is of little value to economics
5. Game theory is of great value to economics
6. The first and second welfare theorems cannot be applied in the real world
7. It is not optimal to assign MWG questions as homework questions

Final Exam (2-3 hrs)

We will have

- ▶ (10% fixed weight) One question on the definitions appeared in lecture notes only
- ▶ These are candidates (You choose three from below)
 - ▶ (min 10%) One question from Ch. 6 Choice under Uncertainty
 - ▶ (min 10%) One question from Ch. 5 Firms
 - ▶ (min 10%) One question from Ch. 3 Classical Demand Theory
 - ▶ (min 10%) One question from Ch.15 or 16 General Equilibrium
- ▶ (min 10%) One question from any chapters
- ▶ At least one of which requires a proof in the answer
- ▶ Anyone of the above will use the Kuhn-Tucker condition
- ▶ I might further refine it and notify you

A test on your understanding of economics

How?

- ▶ Humor has been well defined as thinking in fun while feeling in earnest. (Mark Twain, American novelist)

From JokEC

- ▶ "George Stigler Nobel laureate and a leader of Chicago School was asked why there were no Nobel Prizes awarded in the other social sciences, sociology, psychology, history, etc. "Don't worry", Stigler said, "they have already have a Nobel Prize in ...Literature"

From JokEC

- ▶ "Economics is the only field in which two people can get a Nobel Prize for saying the opposite thing" is true, but is not strong enough. Better:
- ▶ "Economics is the only field in which two people can share a Nobel Prize for saying opposing things." Specifically, Myrdal and Hayek shared one.

From JokEC

- ▶ Economics has gotten so rigorous we've all got rigor mortis.
 - ▶ rigor mortis means the stiffening of the body after death
 - ▶ Presumably said by Kenneth Boulding

From JokEC

- ▶ Heard at the LSE. Econometricians do it if they can identify it.
- ▶ Applied econometricians do it even if they can't.

From JokEC

- ▶ "The only thing more dangerous than an amateur economist is a professional economist."

- ▶ Heard at the Wharton School.
- ▶ Man walking along a road in the countryside comes across a shepherd and a huge flock of sheep. Tells the shepherd, "I will bet you \$100 against one of your sheep that I can tell you the exact number in this flock." The shepherd thinks it over; it's a big flock so he takes the bet. "973," says the man. The shepherd is astonished, because that is exactly right. Says "OK, I'm a man of my word, take an animal." Man picks one up and begins to walk away.
- ▶ "Wait," cries the shepherd, "Let me have a chance to get even. Double or nothing that I can guess your exact occupation." Man says sure. "You are an economist for a government think tank," says the shepherd. "Amazing!" responds the man, "You are exactly right! But tell me, how did you deduce that?"
- ▶ "Well," says the shepherd, "put down my dog and I will tell you."

From JokEC

▶ REASONS TO STUDY ECONOMICS

1. Economists are armed and dangerous: "Watch out for our invisible hands."
2. You can talk about money without ever having to make any.
3. Although ethics teaches that virtue is its own reward, in economics we get taught that reward is its own virtue.
4. When you get drunk, you can tell everyone that you are just researching the law of diminishing marginal utility.

...

From JokEC

Q: How many Chicago School economists does it take to change a light bulb?

A: None. If the light bulb needed changing the market would have already done it.

Q: How many mainstream economists does it take to change a light bulb?

A: Two. One to assume the existence of ladder and one to change the bulb.

Q: How many neo-classical economists does it take to change a light bulb?

A: It depends on the wage rate.

From JokEC

Q: How many Wharton MBAs does it take to change a light bulb?

A: Only one, if you hire me. I can actually change the light bulb by myself. As you can see from my resume, I've had extensive experience changing light bulbs in my previous positions. I've also been named to the Wharton Light Bulb list, and am presently a teaching assistant for Light Bulb Management 666. My only weakness is that I'm compulsive about changing light bulbs in my spare time.

Q: How many B-school doctoral students does it take to change a light bulb?

A: I'm writing my dissertation on that topic; I should have an answer for you in about 5 years.

Q; How many central bank economists does it take to screw in a lightbulb?

A: Just one – he holds the lightbulb and the whole earth revolves around him.

From JokEC

- ▶ A true story:
- ▶ "I heard this from one of my professors. To protect him, no names will be revealed. This professor was about to get married. He went to the jewelers to get a wedding ring for his fiancée. The jeweler told him that he can have the inside of the ring engraved with the name of his fiancée for an additional \$20 (remember, this was a LONG time ago). He said, "But that will reduce the resale value!" The jeweler was aghast. He said, "How can you say such a thing. You are a butcher!" "No," replied the professor, "I am an economist"."

From JokEC

- ▶ A mathematician, a theoretical economist and an econometrician are asked to find a black cat (who doesn't really exist) in a closed room with the lights off:
 - The mathematician gets crazy trying to find a black cat that doesn't exist inside the darkened room and ends up in a psychiatric hospital.
 - The theoretical economist is unable to catch the black cat that doesn't exist inside the darkened room, but exits the room proudly proclaiming that he can construct a model to describe all his movements with extreme accuracy.
 - The econometrician walks securely into the darkened room, spend one hour looking for the black cat that doesn't exist and shouts from inside the room that he has it caught by the neck."