ADV MATERIALS

Prop 3.D.1 If $p \gg 0$, $u(\cdot)$ is continuous, then UMP has a sol'n.

Pf Fact: Weierstrauss Theorem (Math App. M.F): a continuous function on a compact set achieves a maximum. (compact = closed & bounded)

Claim: Walrasian budget set is closed and bounded.

Pf A set in $\mathbb{R}^L$ is bounded iff it is contained inside some ball, or $x_1^2 + \cdots + x_L^2 \leq r^2$

Now, $p_1 x_1 + \cdots + p_L x_L \leq w$

$\Rightarrow x_\ell \leq \frac{w}{p_\ell} \quad \forall \ell$

$\Rightarrow x_1^2 + \cdots + x_L^2 \leq \left(\frac{w}{p_1}\right)^2 + \cdots + \left(\frac{w}{p_L}\right)^2 \leq r^2$

Closed pf:

Let $\lim_{n\to\infty} x_n = x$

$x_n \in B_{p,w}$

Claim: $x \in B_{p,w}$

Suppose $x \notin B_{p,w}$

$\Rightarrow p \cdot x > w \Rightarrow \exists\, \delta > 0, \forall x' \ \|x - x'\| < \delta$ and $p \cdot x' > w$

But since $\lim x_n = x$

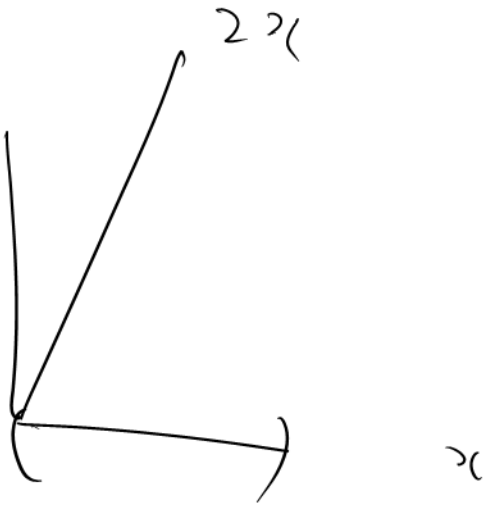
$\Rightarrow \forall \varepsilon > 0 \ \exists N$ s.t. $\forall n > N$, $\|x_n - x\| < \varepsilon$

So when $\varepsilon \leq \delta \Rightarrow \|x_n - x\| < \delta \Rightarrow p \cdot x_n > w$, $x_n \in B_{p,w}$ Contradict

1) violates closedness

Set = $(0, 1)$

$f = 2x$



2) violate boundless

Set = $\mathbb{R}$

$f = 2x$