

# General Equilibrium II

## (Examples)

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## A General Production Model (MWG Section 15.D)

### ▶ Settings

- ▶  $J$  firms
- ▶  $J$  consumer goods
- ▶  $j^{\text{th}}$  firm produces this quantity of a consumer good:  $q_j$
- ▶  $L$  primary inputs (or factors):  $z_j = (z_{1j}, \dots, z_{Lj}) \geq 0$ , which consumers do not consume
- ▶ Factor prices  $w = (w_1, \dots, w_L)$  are strictly positive
- ▶ No intermediate goods (produced goods that are used as inputs)
- ▶ Production function (concave, strictly increasing, differentiable):  $f_j(z_j)$
- ▶ Endowment of factors:  $(\bar{z}_1, \dots, \bar{z}_L)$
- ▶ Firms take prices  $p = (p_1, \dots, p_J) \gg 0$  as given.
- ▶ Output is sold in the world markets
- ▶ Factors are immobile, not traded across countries

## A General Production Model (MWG Section 15.D)

- ▶ Key question:
  - ▶ What are the equilibrium factor prices and allocation of endowments of factors across  $J$  firms?
    - ▶ After this is determined, the UMP can be solved in the standard way we have seen before.

## The Profit Maximization Problem

$$\text{Max}_{z_j \geq 0} p_j f_j(z_j) - w \cdot z_j \quad (1)$$

- ▶ Assuming an interior solution, the necessary and sufficient (since  $f$  is concave) FOCs:

$$p_j \frac{\partial f_j(z_j^*)}{\partial z_{lj}} = w_l^*, \text{ for } j = 1, \dots, J \text{ and } l = 1, \dots, L \quad (2)$$

- ▶ There are  $LJ + L$  endogenous variables because  $(z_1, \dots, z_J) = ((z_{11}, \dots, z_{L1}), \dots, (z_{1J}, \dots, z_{LJ}))$  is in  $\mathbb{R}_+^{LJ}$  and  $(w_1, \dots, w_L)$  is in  $\mathbb{R}_{++}^L$ .
- ▶ But there are only  $LJ$  equations.
- ▶ However, if we add the following  $L$  factor market-clearing conditions, we can solve the system:

$$\sum_j z_{lj}^* = \bar{z}_l \text{ for } l = 1, \dots, L \quad (3)$$

- ▶ What can be learned here?
  - ▶ The main thing is to see how variables (especially factor prices and allocation) are determined endogeneously.
  - ▶ The consumer prices can also be determined endogeneously (Ex. 15.D.4 provides an example but for the less courageous don't attempt to do it as it is difficult)
- ▶ Critique
  - ▶ The system can't be solved if we have one extra endogeneous variable: organizational structure (an example being the production function), which is formed depending on the level of transaction costs